A Course on the History of Mathematics

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Introduction

Since 2005, I have had the opportunity to teach a subject named *History of Mathematics* to undergraduate students at La Trobe University. In 2007, my colleague Grant Cairns and I taught the subject together. A key feature of the subject is that it is based on historical documents rather than a modern text book. The purpose of this paper is to share our experiences in presenting the subject. The terms 'subject' and 'course' are used interchangeably below.

Context

This section describes the context in which the subject was delivered.

Aims of subject

The aims of the subject are described to the students as follows.

> Mathematics is created by human beings and hence is connected with the culture, the times, and the place in which this creative activity takes place. Thus, mathematics has a history — an interesting history. In History of Mathematics, we will focus on mathematics up until the middle ages. We will
> - reflect on mathematical ideas that we met in primary school;
> - encounter some new mathematical ideas;
> - expand our interests in history;
> - study some classic mathematical writings;
> - see mathematics as an intellectual endeavour;
> - gain experience in research, problem solving, and communication.
>
> If you aspire to a teaching career, then the subject will be particularly useful in your professional training.

Pre-requisites

There are no pre-requisite subjects for *History of Mathematics*. We assume only that the students are acquainted with the mathematical ideas encountered at school, namely arithmetic, basic algebra, and some geometry. In particular we do not as-

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sume that the students have ever studied calculus. The subject is an elective subject and is not compulsory in any degree program.

Consequently, the subject is limited to the history of elementary mathematics. We take this limitation even further, and restrict the subject to the history of ancient, elementary mathematics. Such a course ignores the great advances in mathematics since Newton. On the other hand, there is plenty of interesting material to study even with this restriction, especially in a one semester course. It is easier to appreciate the history of a piece of mathematics if one is confident in one's own knowledge of the mathematics itself.

In 2007, the subject was offered to students on the Bundoora and Bendigo campuses of La Trobe University. The students enrolled in the subject came from a range of degree programs. The Bundoora students were enrolled in the Bachelor of Science degrees, although only a couple of them were majoring in the mathematical sciences. Almost all the Bendigo students were undertaking a Bachelor of Education with the intention of becoming mathematics teachers, mainly at the primary or lower secondary levels.

Classes

The course consists of four classes each week for 13 weeks. With small classes, there is no need to distinguish between lectures and tutorials. About one hour each week is devoted to watching a DVD. The DVD documentaries give students a flavour of the culture and times in which the mathematics was created. The DVDs were readily obtained and copies were placed in the library.

Structure, content and resources

History of Mathematics is divided into three approximately equal parts. Each section is based around an original mathematical document (in translation).

Ancient Egypt

The first part of the course deals with the mathematics of ancient Egypt. Our main source of information about the mathematics of ancient Egypt is the *Rhind Mathematical Papyrus* (RMP), which is now housed in the British Museum. Fortunately, the Heyward Library at La Trobe University Bendigo has a copy in [2]. I obtained a copy of the work by the egyptologist Peet [13] through a rare-book dealer. Peet's work is a copy of a facsimile edition of RMP, a translation, and a commentary. Supplementary material was found in [1], [6], [14]. Background material on ancient Egypt was provided in the documentaries [4].

In many text books, the mathematics of ancient Egypt is covered in only a few pages. However, one can study the mathematics in ancient Egypt in some depth by reading RMP for four weeks.

In addition, students learn elementary Egyptian in the process of studying this manuscript. Fortunately, the Egyptian and the mathematics in RMP are fairly simple; the book by Allen [1, Chapters 1 and 9] is an excellent source. Learning from materials in a language other than English is a unique feature of this subject.

Ancient Greece

The second part of the course deals with the mathematics of ancient Greece. We read Euclid's *Elements*. We use Health's translation in [7], and [9]. We cover Book 1, Propositions 1–13 (approximately) of the *Elements*.

The style of this section is good old-fashioned 'theorem and proof' mathematics. Students encounter definitions, axioms, theorems, proofs — and the limitations of Euclid's proofs. Here, students have more exposure to theorems and proofs than in many first year university subjects in mathematics. Students find the material a little dry initially, yet, by the end of this section, they became more critical of the proofs. We hope that they gained admiration for this magnificent intellectual achievement.

There are some flaws in Euclid's proofs. Herein lies another advantage of studying original documents. An error in a modern text book is irritating, whereas finding an error in Euclid is exciting.

Background material on ancient Greece was provided in the documentaries [5].

Medieval Europe

The third section of the course deals with the mathematics of medieval Europe. We read Fibonacci's 13th century work *Liber Abaci* [17]. Fibonacci's aim is to introduce Italians to the Hindu-Arabic numeral system and how it is used in arithmetic. He is hampered by the lack of symbols $+, -, \times, \div, =$ and hence the writing is rather turgid. During four weeks we cover Chapters 1–5. In reading Fibonacci, we encounter his ways of presenting arithmetic and its applications. We find him using 'casting out nines' as a means of checking arithmetical calculations. He introduces us to lattice multiplication that is so often used in schools in the 21st century [17, Chapter 3]. We hope that by the end of this section, students will appreciate the important role that Fibonacci played in the history of mathematics.

Reading Fibonacci was supplemented by the four German documentary programs [3]. The second program in this series highlights the influence of Arab scholars in Spain on science in the middle ages.

Assessment

There are two types of assessment: assignments and an examination.

Assignments

There are two assignments. Each assignment is a 1000-word essay and contributes up to 25% to the final mark in the subject.

The topic of the first essay is 'Who's who in ancient Greek mathematics'. This allows the students the opportunity to supplement their study of Euclid and read about other mathematicians of the period. They may write short paragraphs about many people as in the traditional 'Who's who' format. Alternatively, they may choose to concentrate on just two mathematicians.

The second essay may be on any topic provided that the student discusses it with the lecturers before they embark on the project. At first, students found it difficult to choose a topic and needed guidance. However, they responded very well. They chose topics such as women in mathematics, the history of Sudoku, the origins of algebra, Roman numerals, history of zero, and a description of *The Nine Chapters on the Mathematical Art* [15]. One student gave a 20-minute presentation rather than write an essay. A student who was majoring in mathematics used the opportunity to explore aspects of the Newton–Leibnitz dispute.

Each assignment has a weighting of 25% towards the final mark. Students were given a copy of the marking scheme at the beginning of the semester to encourage certain characteristics in their writing. The marking scheme is in Table 1.

Table 1. Scheme for marking essays

Characteristic	Marks
Is the aim of the essay clearly stated?	2
Is the essay well structured?	3
Is the argument convincing?	4
Is the text linked to the sources?	2
Is there evidence of **your** ideas in the essay?	2
Are sources referenced correctly?	3
Is there variety in types of sources (books, articles, web sites)?	1
Is the language clear?	2
Have you demonstrated some capacity for mathematical word-processing?	1
Is the length of the essay satisfactory?	1
Is the presentation satisfactory?	1
Is the spelling satisfactory?	1
Is the grammar satisfactory?	1
Is punctuation satisfactory?	1
Total	25

Students are asked to demonstrate some capacity for mathematical word-processing because I suspect that, in their university training, students get little experience in this new art. Students who aim to become mathematics teachers will have to set tests and prepare assignments or notes for their students. Students who go on to industry may have to write technical reports. As readers of the *Gazette* know only too well, technical word-processing is non-trivial.

Since prevention is better than cure, I offered to comment on draft copies of the essays, in the second essay in 2007. When reading a draft copy, I tend to give more detailed advice on the work than if I were marking a final paper because I am trying to help the student to improve *this* essay rather than future essays.

Examination

The final examination is a two-hour, closed book examination with no calculator. It has a weighting of 50% towards the final mark in the subject. Below is a sample of questions from the examinations in 2006 and 2007. Attached to the examination was a list of definitions, common notions and the statements of Propositions 1–13 from Euclid's *Elements*, Book 1.

(1) **Ancient numeral systems:**
Construct a multiplication table from 1×1 up to 5×5 using:
   (a) the numeral system of ancient Egypt,
   (b) the numeral system of ancient Greece, and
   (c) the numeral system of ancient Rome.

(2) **Ancient Egypt:**
   (a) How would you multiply by 10 in ancient Egypt?
   (b) Use an example to describe how one would multiply any two natural numbers in ancient Egypt.
   (c) What mathematical principle underpins this method of multiplication?

(3) **Teaching students about numeral systems:**
Describe how you would introduce primary school students to Roman numerals.

(4) **Euclid:**
Using Euclid's approach, prove any two of the first 10 of Euclid's propositions on the attached list according to Euclid's methods.

(5) **Euclid:**
   (a) Explain the meaning of the Latin term 'reductio ad absurdum'.
   (b) Show how Euclid used this method to prove Proposition 6 on the attached list.

(6) **Euclid:**
For Proposition 5 of Book 1 of the Elements, give Euclid's proof in detail. In giving the proof, try to avoid using notions that were not employed by Euclid.

(7) **Medieval multiplication:**
   (a) Describe the method used by Fibonacci in Chapter 2 of *Liber Abaci* to carry out the multiplication 37×49.
   (b) Describe and explain his method for checking the result.

(8) **Medieval division:**
   (a) Describe the method used by Fibonacci in Chapter 5 of *Liber Abaci* to carry out the division of 1346 by 4.
   (b) Describe the method used by Fibonacci in Chapter 5 of *Liber Abaci* to carry out the division of 18456 by 17.
   (c) Briefly, compare Fibonacci's methods with how you might carry out these calculations by hand.

Conclusions

Reading about Euclid and reading the *Elements* are totally different experiences. Approaching the history of mathematics through original materials provides us with intimate connections to the author, and the times and the place in which the mathematical ideas were expressed. This is the principal advantage of the approach. Laubenbacher and Pengelley [11], [12] describe their experience in using this approach for presenting the history of mathematics to students at various levels.

Furthermore, there are additional advantages.

The subject allows students to develop 'graduate attributes' in ways that other mathematical subjects do not. For example, library research, essays, class presentations, and team projects may be incorporated into *History of Mathematics*.

The subject encouraged students to develop their own opinions much more than in other sections of the university mathematics curriculum.

Indeed, in this subject, students with a relatively strong background in mathematics were not especially advantaged. *History of Mathematics* offered so much scope for exploring different aspects of mathematics, that students who are not particularly strong in mathematics could use other skills effectively. Thus, the subject can broaden the appeal of our discipline.

Although the curriculum was restricted to the history of ancient, elementary mathematics, there is a great deal of interesting material available for study. Furthermore, the history of *elementary* mathematics has the potential to be enjoyed by more students than the history of *advanced* mathematics.

One could vary the course by using other original mathematical documents. The works by Archimedes, Appolonius of Perga, and Nicomachus are available in [8]; note that this version of Archimedes' work is a modernised version of the works of Archimedes rather than a faithful translation. The first volume of a faithful translation of the works of Archimedes has been produced by Netz [10]. A section on the mathematics of China could be based on the classic work [15]. Fibonacci's *Book of Squares* [16] would provide an excellent introduction to number theory.

We often hear the argument in universities that mathematicians should teach mathematics subjects. Who should teach *History of Mathematics* — a mathematician or a historian? By studying original mathematical manuscripts, the subject has a very strong mathematical focus and hence, with this approach, it is reasonable, that the subject be taught by a mathematician.

The documentaries on the DVDs gave students a feel for the place and times in which the mathematics was created. These programs linked our subject with other disciplines such as archaeology, ancient and medieval history, and the history of science, technology, architecture, building, and agriculture.

The students did learn some *mathematics* in this subject. They did not know about Egyptian fractions before undertaking this subject. Some had encountered Fibonacci only in connection with Fibonacci numbers. Most students knew nothing about Euclid at the beginning of the course, and had little prior experience in proving theorems until they immersed themselves in the *Elements*. None knew about casting out nines before the course.

Furthermore, students developed a clearer appreciation of things that we take for granted in mathematics. For example, Fibonacci shows us the importance of place value in our number system.

More importantly, *History of Mathematics* provides new and different ways to gain confidence in mathematics, and is a bridge between 'the two cultures'.

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History Topics Index. Mathematics starts with counting. It is not reasonable, however, to suggest that early counting was mathematics. Only when some record of the counting was kept and, therefore, some representation of numbers occurred can mathematics be said to have started.Â Of course ax = b contains other conventions of notation which we use without noticing them. For example the sign "=" was introduced by Recorde in 1557.Â We view the history of mathematics from our own position of understanding and sophistication. There can be no other way but nevertheless we have to try to appreciate the difference between our viewpoint and that of mathematicians centuries ago. Often the way mathematics is taught today makes it harder to understand the difficulties of the past. The area of study known as the history of mathematics is primarily an investigation into the origin of discoveries in mathematics and, to a lesser extent, an investigation into the mathematical methods and notation of the past. Before the modern age and the worldwide spread of knowledge, written examples of new mathematical developments have come to light only in a few locales. From 3000 BC the Mesopotamian states of Sumer, Akkad and Assyria, together with Ancient Egypt and Ebla began using arithmetic