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ON THE CARLEMAN FORMULA FOR MATRIX BALL

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Survey of known results on the Carleman formula can be found in the book by L.A. Aĭzenberg [1]. In particular, in homogeneous domains in $\mathbb{C}^n$ for finding such formulas one can use the automorphism groups (see [1], Chap. 6). In [2] the case of the Siegel domains, i. e., unbounded realizations of homogeneous domains, was considered and the Carleman formulas which reconstruct the values of holomorphic functions on a skeleton of a Siegel domain (not in proper domain) were given. The Carleman formula for a function of matrices was cited in [3].

In the present article we consider a matrix ball $\mathfrak{B}$. Using the properties of the Bergman, Szegó, and Poisson kernels for $\mathfrak{B}$ from [4], we find the Carleman formula which reconstructs the value of a holomorphic function in the domain $\mathfrak{B}$ by the values on a part of the boundary.

Let $Z = (Z_1, Z_2, \dots, Z_n)$ be a vector composed of square matrices Z_j of order m, which are considered over the field of complex numbers $\mathbb{C}$. We may assume that Z is an element of the space $\mathbb{C}^{m^2 n}$. In this set of vectors we introduce a matrix "scalar" product

$$\langle Z, W\rangle = Z_1 W_1^* + \dots + Z_n W_n^*,$$

where W_j^* is a matrix conjugate and transposed for the matrix W_j.

Consider in the space $\mathbb{C}^{m^2 n}$ the following domain

$$\mathfrak{B} = \{Z : E^{(m)} - \langle Z, Z\rangle > 0\}, \tag{1}$$

where $E^{(m)}$ is the unity matrix of order m, which is called a matrix ball. The skeleton of this domain is the variety

$$\Delta_{\mathfrak{B}} = \{Z : \langle Z, Z\rangle = E^{(m)}\}; \tag{2}$$

here $\mathfrak{B}$ is a bounded complete circular domain.

We shall study automorphisms of the matrix ball $\mathfrak{B}$, which send arbitrary point $B \in \mathfrak{B}$ to the origin. These holomorphic automorphism groups were described in [4].

Theorem 1 (see [4]). *In order for a mapping of the form*

$$W_k = R^{(-1)}(E^{(m)} - \langle Z, B\rangle)^{-1} \sum_{s=1}^{n} (Z_s - B_s) Q_{sk}, \quad k = 1, \dots, n \tag{3}$$

to be an automorphism of the matrix ball it is necessary and sufficient that the matrices R, Q_{sk}, $s, k = 1, \dots, n$, satisfy the relations

$$R^*(E^{(m)} - \langle B, B\rangle)R = E^{(m)},$$
$$Q^*(E^{(mn)} - B^* B)Q = E^{(mn)},$$