



INTRODUCTION TO FOURIER ANALYSIS

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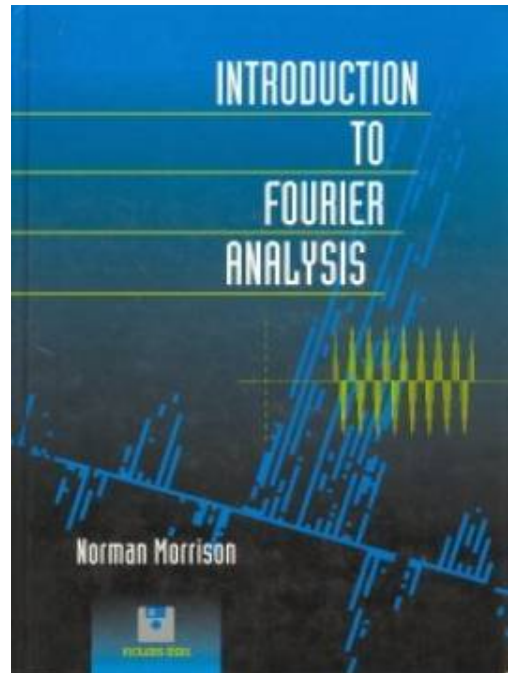
SUMMARY

Comprehensive, user friendly, and pedagogically structured

A fast, easy way to learn about the electrical engineer's most important mathematical tool

Based on a groundbreaking one-semester course originated by Professor Norman Morrison at the University of Cape Town, this book serves equally well as a course text and a self-study guide for professionals. Offering only relevant mathematics, it covers all the core principles of electrical engineering contained in Fourier analysis, including the time and frequency domains; the representation of waveforms in terms of complex exponentials and sinusoids; complex exponentials and sinusoids as the eigenfunctions of linear systems; convolution; impulse response and the frequency transfer function; magnitude and phase spectra; and modulation and demodulation.

- Covers Fourier analysis exclusively for electrical engineering students and professionals
- Offers a complete FFT system Contained on the enclosed disks long for IBM compatibles, the other for Macintosh)
- Includes dozens of examples drawn from electrical engineering
- Packed with exercises, samples, and end-of-chapter problem sets



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Chapters 16 and 17 (Located in README files on the disks)

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In mathematics, Fourier analysis (sometimes called harmonic analysis) is the study of the way general functions may be represented or approximated by sums of simpler trigonometric functions. Fourier analysis grew from the study of Fourier series, and is named after Joseph Fourier, who showed that representing a function as a sum of trigonometric functions greatly simplifies the study of heat transfer. Introduction to the Fourier analysis of functions. Journal of Analyse Mathématique, Vol. 7, Issue. 1, p. 281. This monograph on generalised functions, Fourier integrals and Fourier series is intended for readers who, while accepting that a theory where each point is proved is better than one based on conjecture, nevertheless seek a treatment as elementary and free from complications as possible. Little detailed knowledge of particular mathematical techniques is required; the book is suitable for advanced university students, and can be used as the basis of a short undergraduate lecture course. A valuable and original feature of the book is the use of generalised-function theory to derive a simple, wide